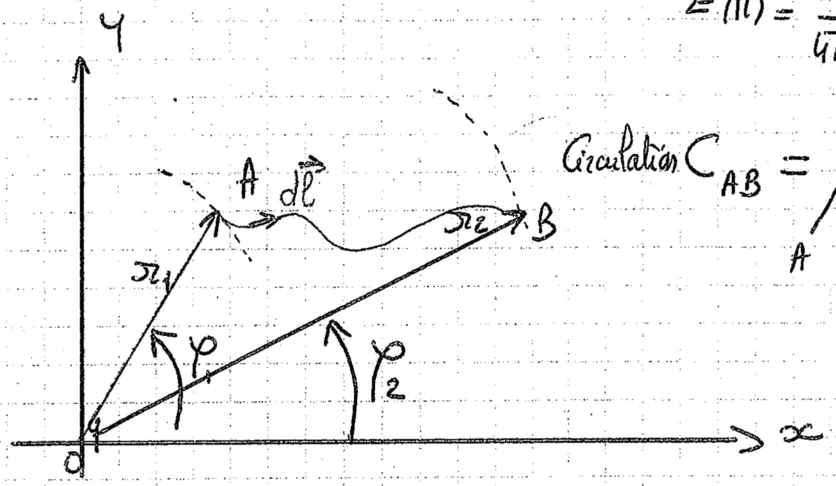
Différence de potentiel entre deux points

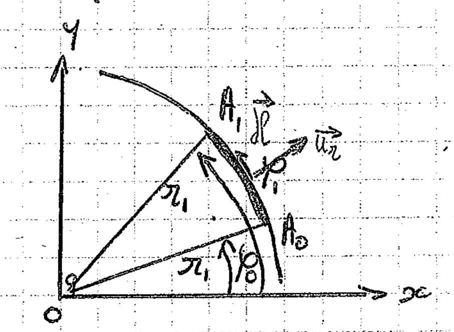
$$\vec{E}(M) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{u}_r$$

1)



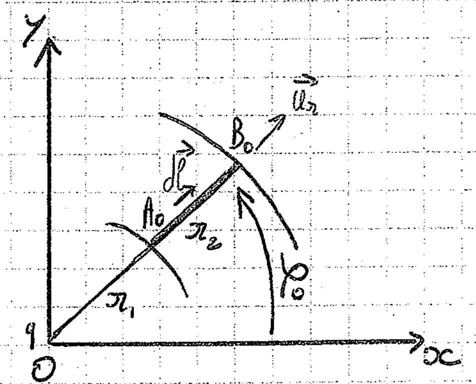
Circulation $C_{AB} = \int_A^B \vec{E}\cdot d\vec{l}$

a) $r_1 = r_2$ on se déplace sur le m̂ cercle.



$$\underset{(\vec{u}_r)}{\vec{E}} \perp d\vec{l} \Rightarrow \vec{E}\cdot d\vec{l} \equiv 0 \quad \forall M$$
$$\Rightarrow C_{A_0A_1} = 0$$

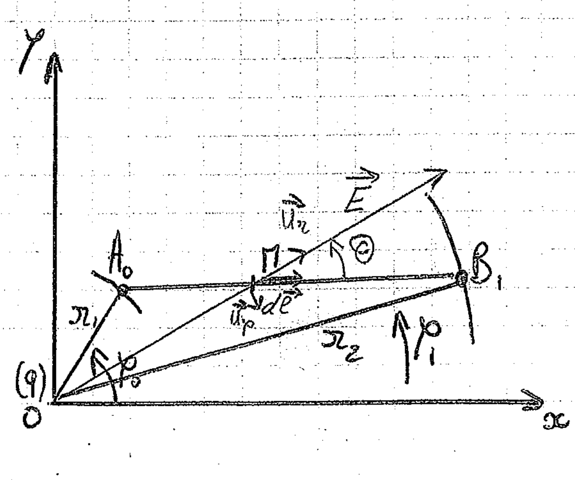
b)



$$\underset{(\vec{u}_r)}{\vec{E}} \,//\, d\vec{l} \Rightarrow \vec{E}\cdot d\vec{l}$$

$$C_{AB} = \int_{r_1}^{r_2} E(r)\,dr = \frac{1}{4\pi\varepsilon_0}\, q \int_{r_1}^{r_2} \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

c)



dc élément infinitésimal de circulation

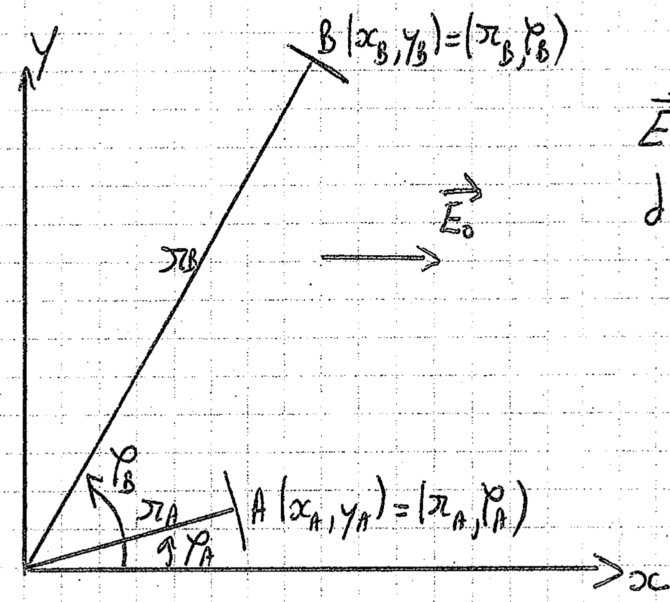
$$dc = \vec{E}\cdot d\vec{l}$$

avec $d\vec{l} = dr\,\vec{u}_r + r\,d\varphi\,\vec{u}_\varphi$
or $\vec{E} \,//\, \vec{u}_r$ et $\vec{E} \perp \vec{u}_\varphi$

donc $dC = \vec{E}\cdot d\vec{l} = \vec{E}\cdot(dr\,\vec{u_r} + r\,d\varphi\,\vec{u_\varphi})$
$= E(r)\,dr$

d'où $C_{A_0B_1} = \int_{r_1}^{r_2} E(r)\,dr = \frac{q}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

2/



$\vec{E} = E_0\,\vec{e_r}$

$dC = \vec{E_0}\cdot d\vec{l}$ (coordonnées cartésiennes)

$= E_0\,\vec{e_r}\cdot(dx\,\vec{e_x} + dy\,\vec{e_y})$

$= E_0\,dx$ $\quad \vec{e_r} \perp \vec{e_y}$

coordonnées cartésiennes il vient $V_B - V_A = \int_A^B dC = E_0\int_{x_A}^{x_B} dx = E_0(x_B - x_A)$

coordonnées cylindriques $x_A = r_A\cos\varphi_A$ et $x_B = r_B\cos\varphi_B$

d'où $V_B - V_A = E_0(r_B\cos\varphi_B - r_A\cos\varphi_A)$

or $V_A - V_B = -(V_B - V_A) = E_0(r_A\cos\varphi_A - r_B\cos\varphi_B)$